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SUMMARY REPORT OF RESULTS, CONCLUSIONS AND RECOMMENDATIONS FROM--ETC(U)

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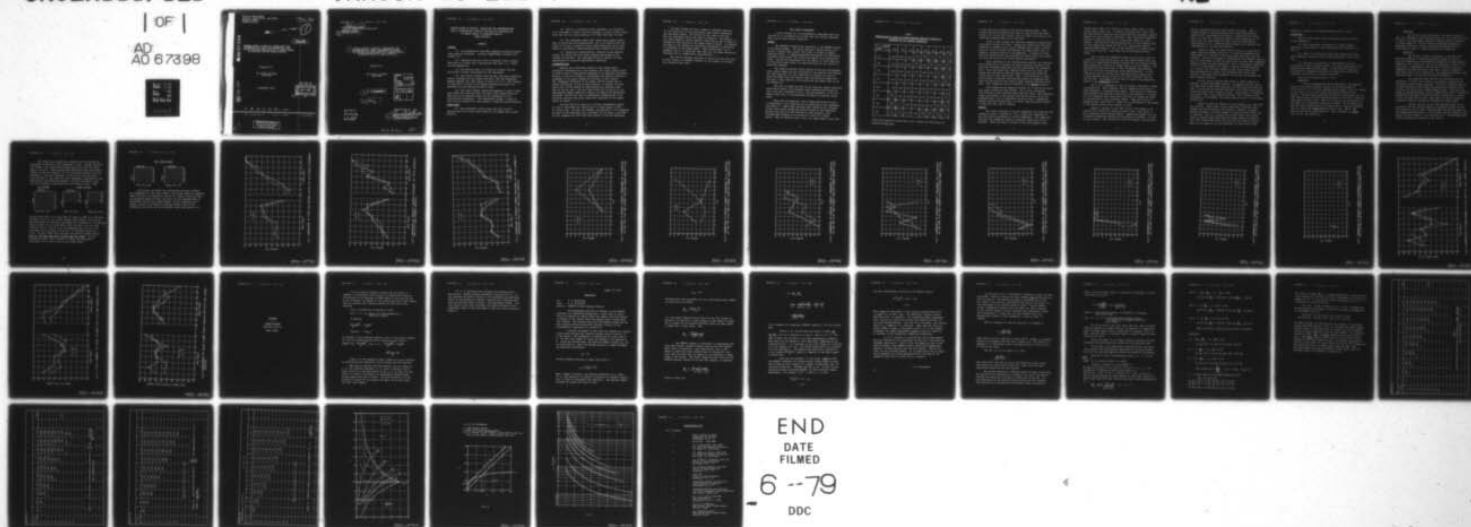
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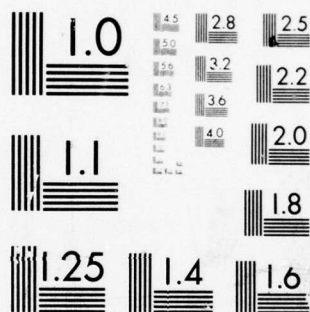
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SUMMARY REPORT OF RESULTS, CONCLUSIONS AND
RECOMMENDATIONS FROM A PSYCHOPHYSICAL STUDY
OF THE RELATIVE DETECTABILITY OF TARGET TRACKS
IN SIMULATED PASSIVE SONAR DISPLAYS

Prepared for

The Bureau of Ships
Code 689B

6 September 1963

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SUMMARY REPORT OF RESULTS, CONCLUSIONS AND RECOMMENDATIONS
FROM A PSYCHOPHYSICAL STUDY OF THE RELATIVE DETECTABILITY
OF TARGET TRACKS IN SIMULATED PASSIVE
SONAR DISPLAYS

ABSTRACT

RESULTS

(1) A psychophysical experiment employing simulated passive sonar data was performed. (Contract NObsr-85185, Problem 8 and Problem 13.)

(2) Identical data were used to construct visual display materials in a stereo and a conventional flat form, i.e., three- and two-dimensional.

(3) Ten subjects (Ss), all young men students from The University of Texas, participated in the experiment.

(4) The results indicate that the best detection capability exists, averaged over all signal densities, when the background masking density (for noise alone) is from 0.5 to 0.7. This finding is very useful in that it will permit threshold settings to be specified to yield maximum detection rates.

(5) The stereo display tentatively appears to yield a lower false alarm rate than the flat, and those false alarms which are made tend to be rejected earlier but the difference is not statistically significant. The experimental design, it must be said, was not sensitive to false alarm or rejection characteristics.

CONCLUSIONS

(1) This experiment clearly shows that the over-all marking density for noise alone should be set higher than current practice.

(2) There is, according to this experiment, an indication of an advantage of stereo over flat displays in terms of false alarm rate; the rate for stereo tending to be lower than that for flat.

(3) The experiment showed that tracks were equally detectable in stereo and flat presentations, but it is believed that the result is in part a function of poor depth definition, and in part owing to the method of apportioning the data between the two eyes.

(4) Theoretical considerations, detailed in the Appendix, support the belief that the stereo display should show an advantage in detection. The advantage should be capable of being demonstrated in corrected experimental conditions.

RECOMMENDATIONS

→ Because the stereoscopic method of data display has a theoretical advantage over a flat display, the stereo method should be shown to have a demonstrable advantage in an empirical experiment. Failure of the first attempt to demonstrate the expected detection advantage of the stereo display we attribute to four causes:

(a) Inadequate definition of the depth; (b) Not enough data to obtain reliability; (c) No accurate measure, or control, of false alarm rate; (d) The method of apportioning the simulated sonar data to the two eyes. The last three items are a function of the experimental design, which was supposed to simulate an operational situation. We now feel that an experimental design aimed at the detection function with false alarm rate controlled is the proper course to pursue. *next page*

The data should be taken in a stricter experimental manner, if only to facilitate the acquisition of a large amount of data in a short time. We were, for example, getting about 4 to 6 judgments per hour in the experiment just concluded, but we expect to get 200 or more judgments per hour from each subject in the new experiment.

→ We shall design and test a number of different patterns for the definition of the stereoscopic space before commencing the experiment proper. The pattern we have used in the past does not adequately define the plane of interest: the plane is not easily apparent, and an observer finds that he has to "work at" keeping his eyes pointed at the proper plane of depth. In addition, the edges of the display are peculiarly susceptible to difficulty of perception in the proper plane. These difficulties can be remedied, and it is hoped that when they are, the known theoretical advantage will be demonstrable experimentally. ←

The lack of definite advantage of stereo over flat was also, we feel, owing to a drawback inherent in the scheme for printing the simulated sonar data.

THE SCENICS EXPERIMENT

A brief outline of the psychophysical experiment which was carried out is included here in order that the results discussed later may be more readily understood.

METHOD

The apparatus required for simulation of the SCENICS display is relatively simple. The method employed a motion picture projector, a ground glass screen, and a pair of prisms to achieve binocular fusion. A computer generated and printed the data which simulated "noise alone" and "noise-plus-signal" in an appropriate track. Control was maintained over the density of noise marks and of noise-plus-signal marks, and means provided for the control of binocular parallax.

The computer-printed visual display fields were photographed with a movie camera and were projected to the observers who advanced the films manually, but were instructed to operate at a rate of about one frame every two seconds. Each new projected frame had a new line of data entered at the bottom with the oldest line at the top dropped off.

Two sets of tracks for each of the noise and signal conditions indicated in Table I were prepared. Each condition was printed in such a way that, depending on how it was photographed, it was displayed either flat or with parallax introduced to generate visual depth.

There were 10 observers, each of whom viewed the 78 stereo (3-dimensional) film strips for the 39 conditions of the experiment (see Table I). These same observers also viewed 39 flat (2-dimensional) film strips during the control portion of the experiment. Records were kept for each observer in each condition, thus allowing each response to be scored as a "hit" or a "false alarm." When an observer made a call, the frame number of the film

TABLE I
COMBINATIONS OF SIGNAL AND NOISE MARKING DENSITIES PRINTED BY
COMPUTER FOR THE SCENICS SIMULATION

SIGNAL NOISE	.05	.10	.15	.20	.25	.30	.35	.40
.10	3.52	6.02	7.96					
.20	1.93	3.52	4.86		7.04			
.30	1.33	2.50	3.52	4.44	5.26	6.02		
.40	1.04	1.93	2.76	3.52	4.22	4.86		
.50	.83	1.58	2.28	2.92	3.52	4.08	4.61	5.10
.60		1.34	1.93	2.50	3.01	3.52	4.00	
.70			1.67	2.18	2.65	3.10		
.80			1.49	1.93				

Cells with numerical entries were used, crossed out cells were not.
 Entries are $\frac{S}{N}$ ratios.

at that time was recorded as well as the bearing called. This allowed scoring of responses not only in terms of hits and false alarms, but, in the case of hits, also in terms of how much of the track was displayed when the hit was made.

Prior to data taking, the observers underwent a familiarization period of about one week. During this period they were given knowledge of whether or not their responses were correct, various hints as to how to obtain and maintain the depth effect, and, in general, received practice on the apparatus. It should be noted that all observers said they obtained the depth effect very easily, and they did so with very little special instruction.

Upon completion of the familiarization period, the observers were divided into two groups; one group receiving one-half of the 3-D films (randomly presented) and the other group, the remaining half. After each group had completed these first 39 films, the groups were switched so that each group received the films previously seen by the other group. Thus, it was possible to obtain some estimate of practice and learning effects by comparing data from the first half of the films to those obtained from the second half. This comparison will be discussed in the RESULTS section.

When observers had completed viewing all of the 78 3-D films, they began viewing the flat display. Due to time limitations, only one-half of the available flat films were used. This procedure seemed justifiable on the grounds that the two halves of the 3-D experiment gave rise to essentially identical data. Comparisons between flat and 3-D data were, of course, made with equal amounts of data from each type of display.

RESULTS

The results obtained in the two experiments are presented and compared in Figs. 1 through 14. Fig. 1 shows the comparison of the first and second halves of the 3-D experiment. The comparison indicates the negligible effects of practice during the 3-D experiment. This, and each of several of the other graphs, plots

percent hits, $P(H)$, as a function of noise density (Fig. 1a) and signal density (Fig. 1b). To obtain the percent hit, $P(H)$, value, the data from all observers were pooled and the percentage computed against the total number of observers. Thus, the points represent percent of observers who hit on a particular signal (or noise) density. The function relating percent hits to noise density (Fig. 1a) was obtained by adding all hits for a given noise density, ignoring signal density. Fig. 1b was obtained in a similar manner. It is clear from Fig. 1 that there was no detectable practice effect.

Fig. 2 shows the comparison between 2-D and 3-D of $P(H)$ as a function of noise density (Fig. 2a) and signal density (Fig. 2b). There is no significant difference between the two functions either when plotted as a function of signal density or noise density. One interesting result shown here is that the highest detectability does not occur with the smallest noise marking density. In fact, $P(H)$ may be seen to increase with increasing noise density up to a noise marking density of 0.60 in the case of 2-D and 0.70 for the 3-D case. An attempt will be made to explain this unexpected result in the DISCUSSION section.

Fig. 3 is a comparison of the two display techniques in terms of percent hits. Here, however, $P(H)$ is computed on the basis of total responses - including false alarms. This method of computing $P(H)$ decreases the apparent sensitivity of the observers, of course, but also indicates that false alarm rate does not change the shape of the functions in any systematic manner. This may be seen by comparing Fig. 2 with Fig. 3. There are no differences in the shapes of the function which would indicate a trend.

Figures 4 through 11 plot $P(H)$ (out of total number of observers) as a function of signal-to-noise ratio. Each figure is for a different noise marking density. Again there appear to be no consistent differences between the two methods of display. It will be seen, however, that for both 2-D and 3-D displays, the functions

have an increasing slope as noise density increases. This is another example of the interesting result that detectability improves with increasing noise density. Another way of putting this is to say that, within the range of noise densities, up to about 0.70, the same signal-to-noise ratio will yield better detectability if it is achieved with a higher noise density, and complementarily, the same detectability.

Figure 12 shows the frame number of hits as a function of signal and noise marking densities. It will be remembered that frame number is a measure of the track length when the track was detected. Owing to the method of obtaining the 3-D display, however, the 3-D track length is half as long as the 2-D track when comparable amounts of data are being presented. Thus, a legitimate comparison is not of track lengths, but rather of amount of data presented. As a result, the 3-D track lengths of Fig. 12 have been doubled. The data shown here requires the conclusion that, when equal amounts of data have been presented, the 3-D and 2-D displays are about equally effective.

In Fig. 13 are plotted total number of false alarms as a function of noise and signal density. It appears here that there are fewer false alarms made under the 3-D conditions than under the 2-D. It will also be seen that observers do not false alarm under conditions which lead to high hit rates. This result is a function of the fact that if the observer has made a call in which he has confidence it is probably a hit and he does not give it up and make another call.

Figure 14 shows the lengths of false alarms at the time they are rejected. There seems to be no particular trend when these lengths are plotted against noise density. However, as signal density increases, the track lengths of rejected false alarms tend to decrease. This is a function of the fact that a good (easy) track is usually found early when the signal density is high and thus false alarms made before this time are also rejected early. There do not appear to be any differences in false alarm track

lengths as a function of display technique (2-D vs. 3-D).

CONCLUSIONS

There are three main points which arise from the results of the experiment.

(1) Best detections, irrespective of signal marking densities used are made at noise marking densities between 0.5 and 0.7.

(2) There is an indication that false alarms are called less frequently, and when called rejected earlier, in the stereo case.

(3) The lack of difference in detectability is to be attributed to three principal causes: (a) poor structuring of the plane-in-depth where tracks appear, (b) the method of printing the blocks of simulated sonar data, (c) There were insufficient judgments from subjects to obtain reliability.

Let us take up these points in turn.

Point (1)

From the mathematical theory developed in the Appendix there are no readily apparent reasons for best detections to occur in the 0.5 to 0.7 noise marking density range. There is nothing in the theory, on the other hand, which would contradict the results. We would argue qualitatively that best detections occur in that range because at lower marking densities, even with a relatively high signal-to-noise ratio, marks in a track are too scattered, too far apart, to be discerned visually as related. On the other hand, when the noise marking density is higher than 0.7 the best possible (for our conditions) input signal-to-noise ratio, $\frac{\Delta p}{p}$ is $\frac{.2}{.8}$, which is too low for good detection. This represents an output $S/N = 0.749$. (See Appendix).

Point (2)

We can only state tentatively that false alarms are made less frequently, and rejected earlier, in the stereo case from our data. The experimental design we employed was not sensitive to false alarm rate. We are therefore proposing elsewhere an experiment which will correct this defect by making use of a detectability index, d' , which controls false alarm rate.

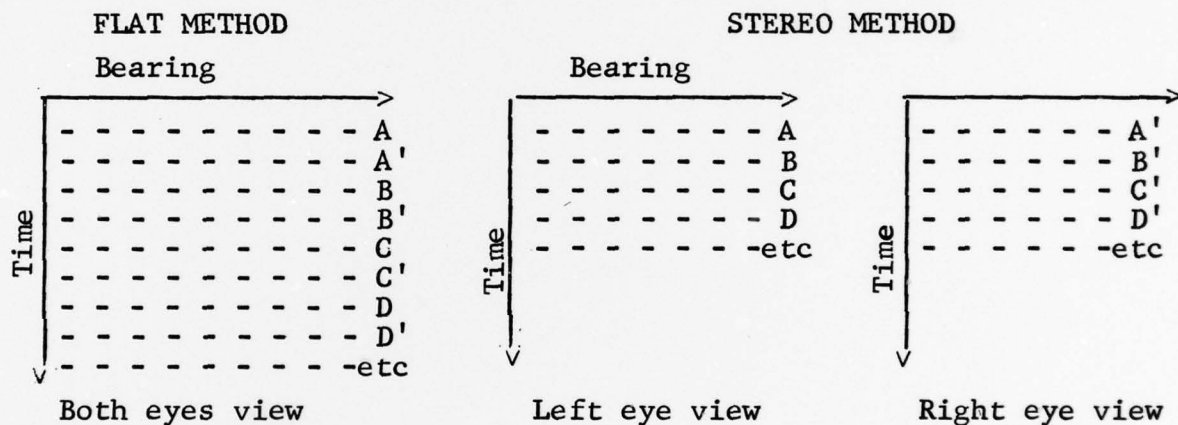
Point (3)

The failure to show a difference in detection rate which was expected on theoretical grounds stems, we believe, from the drawbacks of an experimental design which, when proposed, appeared to be a valid one. One of its drawbacks was that a very great deal of experimental time was used to obtain a very few judgments: the data rate was so low that it was not feasible to take enough data to get reliability. Any expected difference would therefore tend to be obscured by subject variability.

More important than the lack of sufficient data for reliability, we feel, was the poor structuring or definition of the plane in which targets appeared. It will be recalled from our original proposal how the plane-of-interest was indicated, with a pattern of circles and a diamond-shaped figure. This left some areas not adequately marked, and so subjects were made to "work at" finding the right plane-in-depth instead of being able to attend to the detection task as such. This defect is easy to remedy by choosing a different pattern for defining the proper plane.

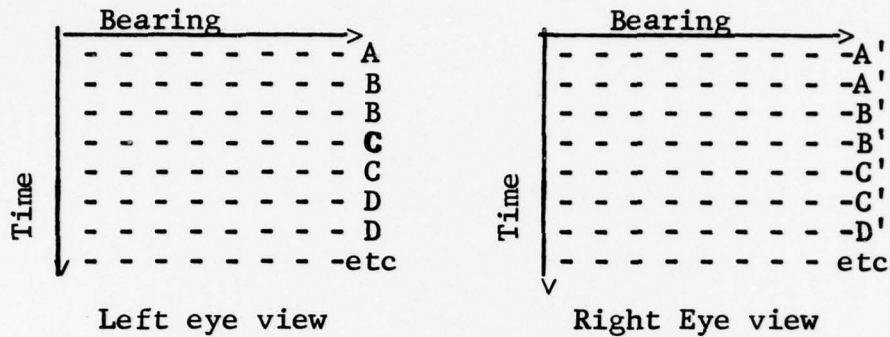
Lastly, and this is the most important point of all, we believe that the results showed no difference between 3-D and 2-D because we were in effect throwing away one-half the pairs of the signal-plus-noise marks. Let us examine how we printed out the blocks of data, briefly, to see why this is so. (The Appendix also contains information relating to this point).

The display area simulated a bearing-time plot having 100 increments of bearing by 220 increments of time. Marking decisions were binary: for a particular bearing-time bin a mark was either made or not made. These bearing-time dimensions were used without change for the plot display, but for the stereo display every odd increment of time was displayed to the left eye only and every even increment to the right eye only, producing a binocularly fused display area 100 bearing increments x 110 time increments, with each time increment containing twice the amount of data. Thus:



In the stereo case it is clear that if a mark is made in an odd time increment, say in A, at a given bearing and a mark is also made in the same bearing on the next succeeding time increment, A', a binocular fusion results. In the case where a mark is made in an even time interval, say B', and not in B, the preceding one, but in C, the succeeding one (as before in A and A') no binocular fusion can occur. We thus "threw away" one-half the possible signal fusions. Even so, the flat case was no better than the stereo. We must reintroduce these "lost" fusions to realize the theoretical advantages, and it can be done by the scheme shown:

NEW STEREO METHOD



From points (2) and (3) the conclusions are clear: perform the experiment in a way that yields a very large number of judgments, which will give us reliability, and do it so that it does not take an inordinate amount of time; and print out the simulated data in accordance with the new stereo method. We are proposing such a design, and we do so because we strongly believe that a real difference does exist and the difference needs being made use of.

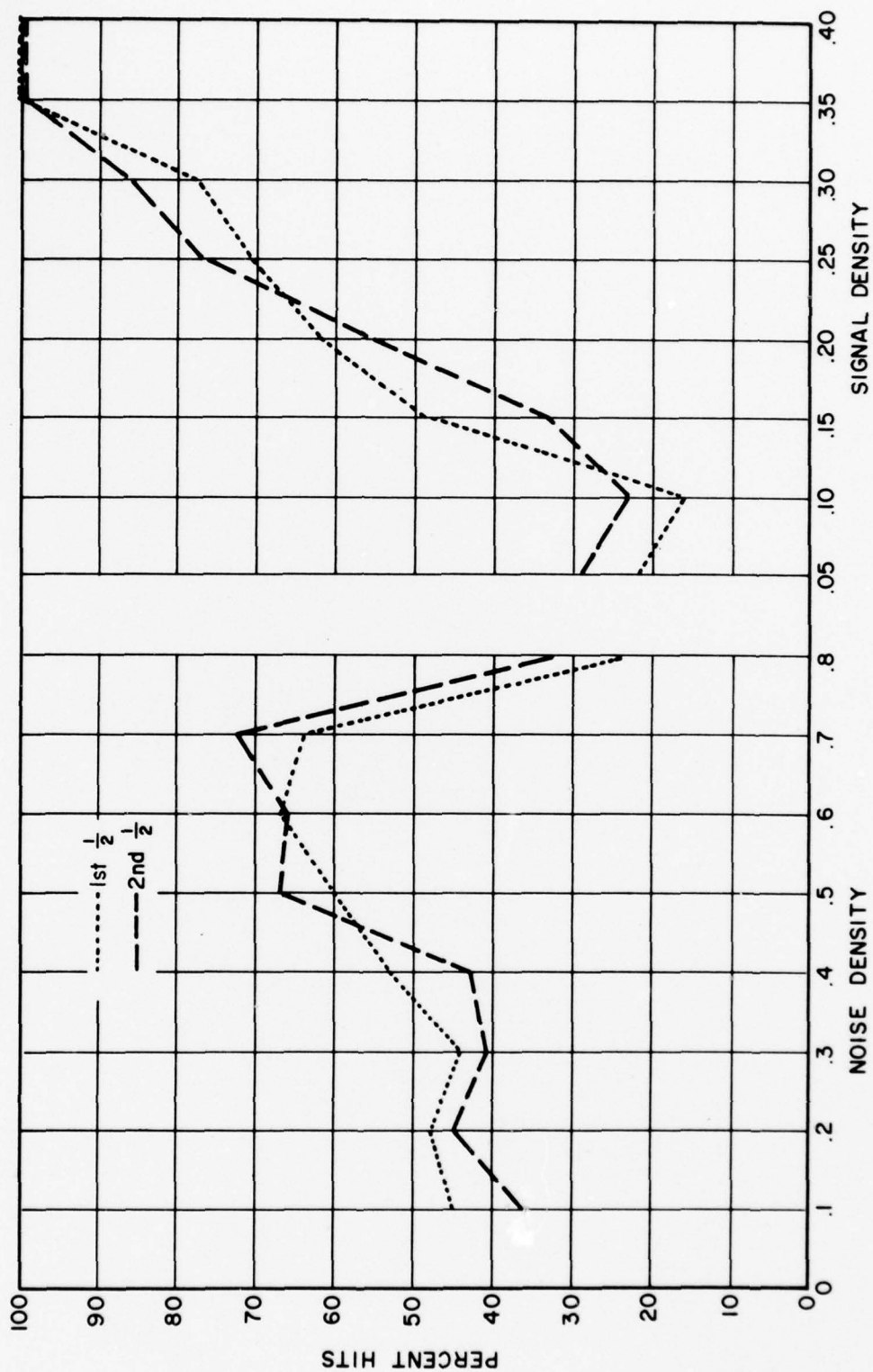


Fig. 1 - COMPARISON OF FIRST AND SECOND HALVES OF THE STEREO EXPERIMENT

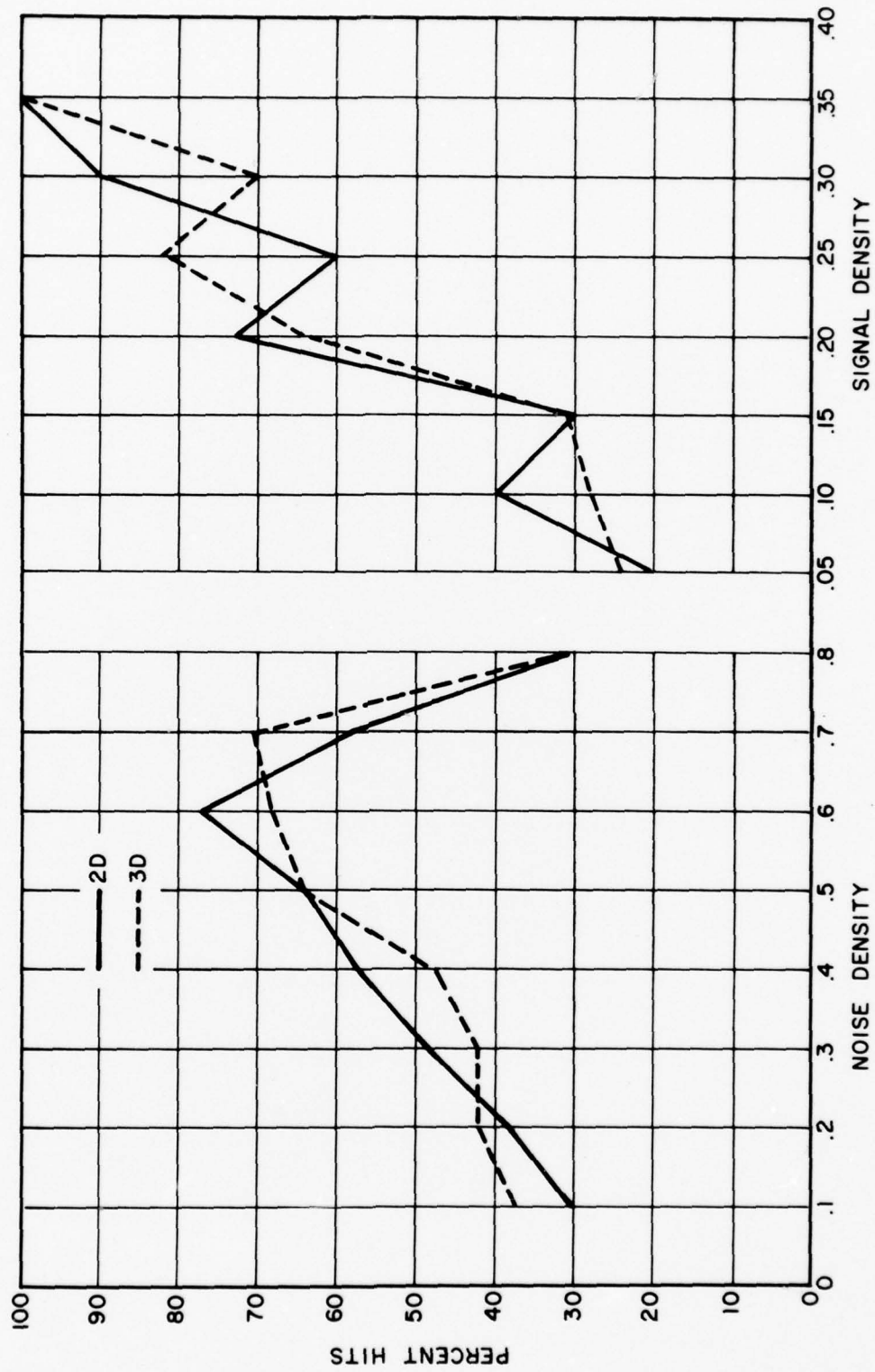


Fig. 2 - COMPARISON OF PERCENT CORRECT RESPONSES (OF TOTAL SUBJECTS) FOR FLAT AND STEREO DISPLAYS

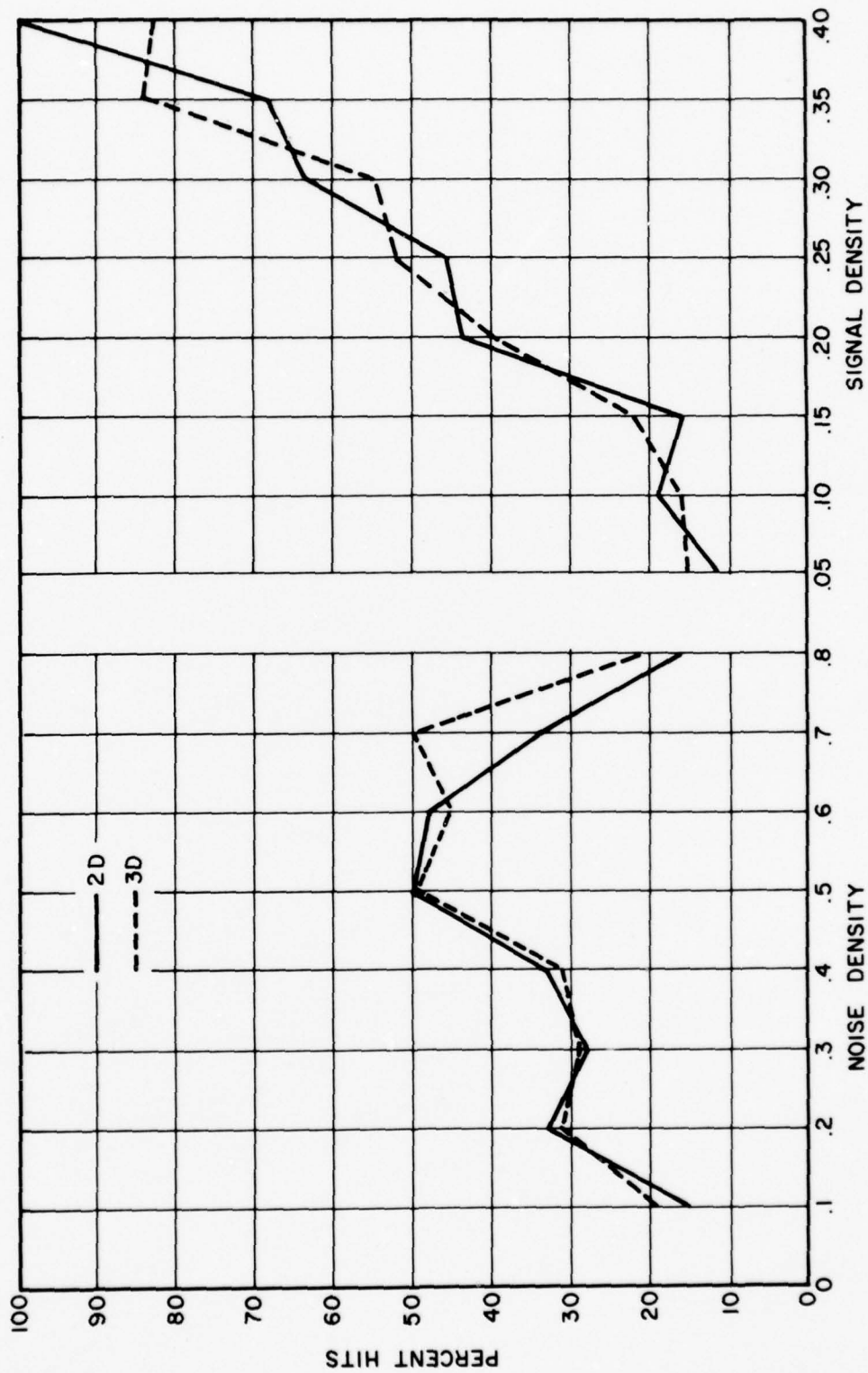


Fig. 3 - COMPARISON OF PERCENT CORRECT RESPONSES (OF TOTAL NUMBER OF RESPONSES) FOR FLAT AND STEREO DISPLAYS

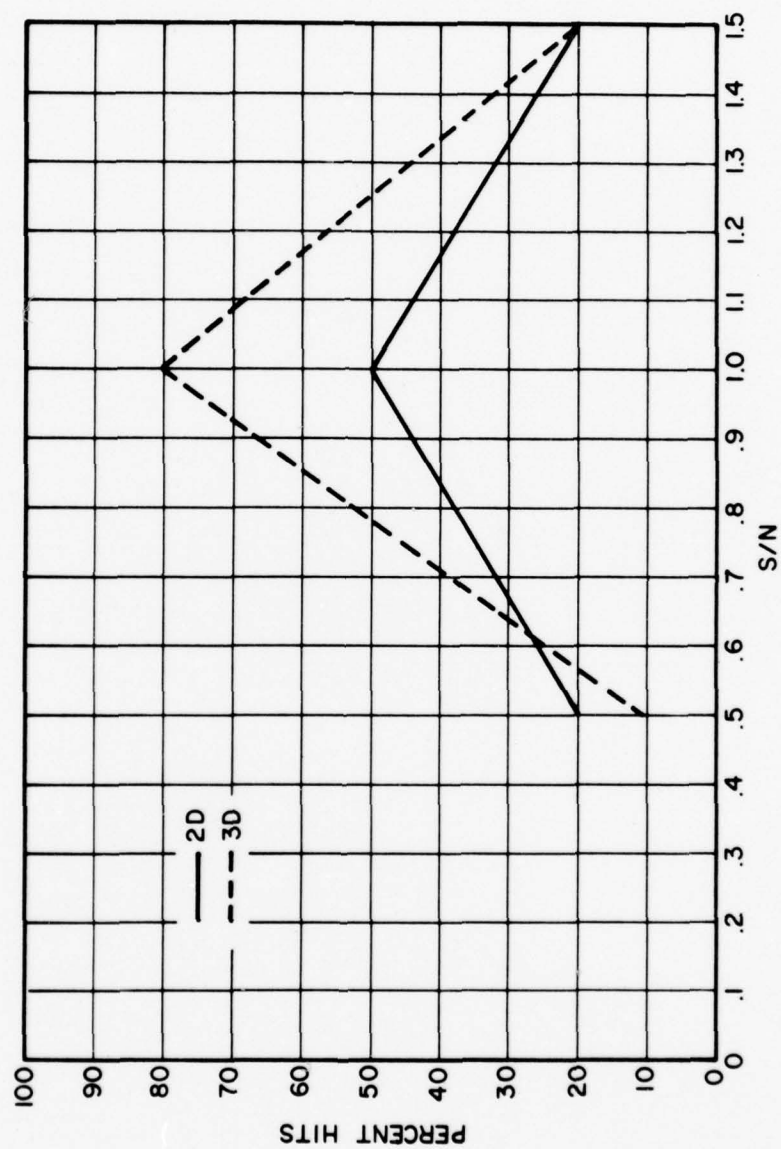


Fig. 4 - COMPARISON OF PERCENT CORRECT RESPONSES AS A FUNCTION OF S/N FOR FLAT AND STEREO DISPLAYS. (NOISE DENSITY = 0.10)

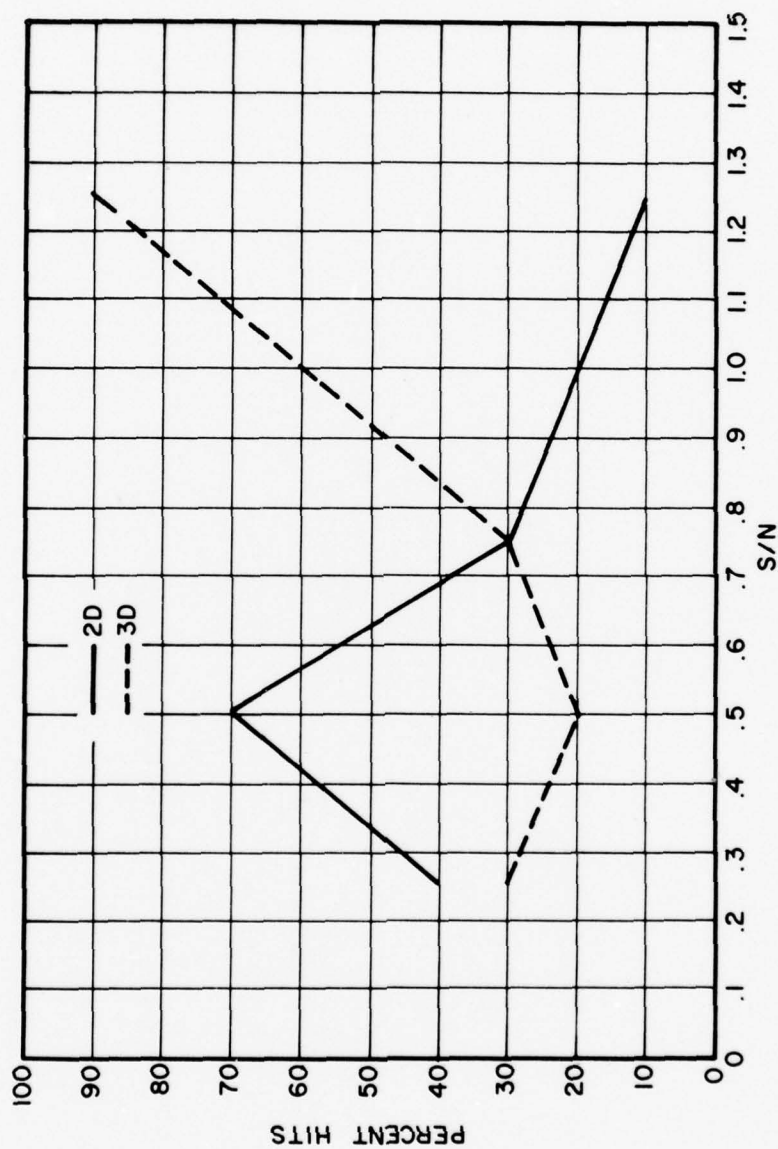


Fig. 5 - COMPARISON OF PERCENT CORRECT RESPONSES AS A FUNCTION OF S/N FOR FLAT AND STEREO DISPLAYS. (NOISE DENSITY = 0.20)

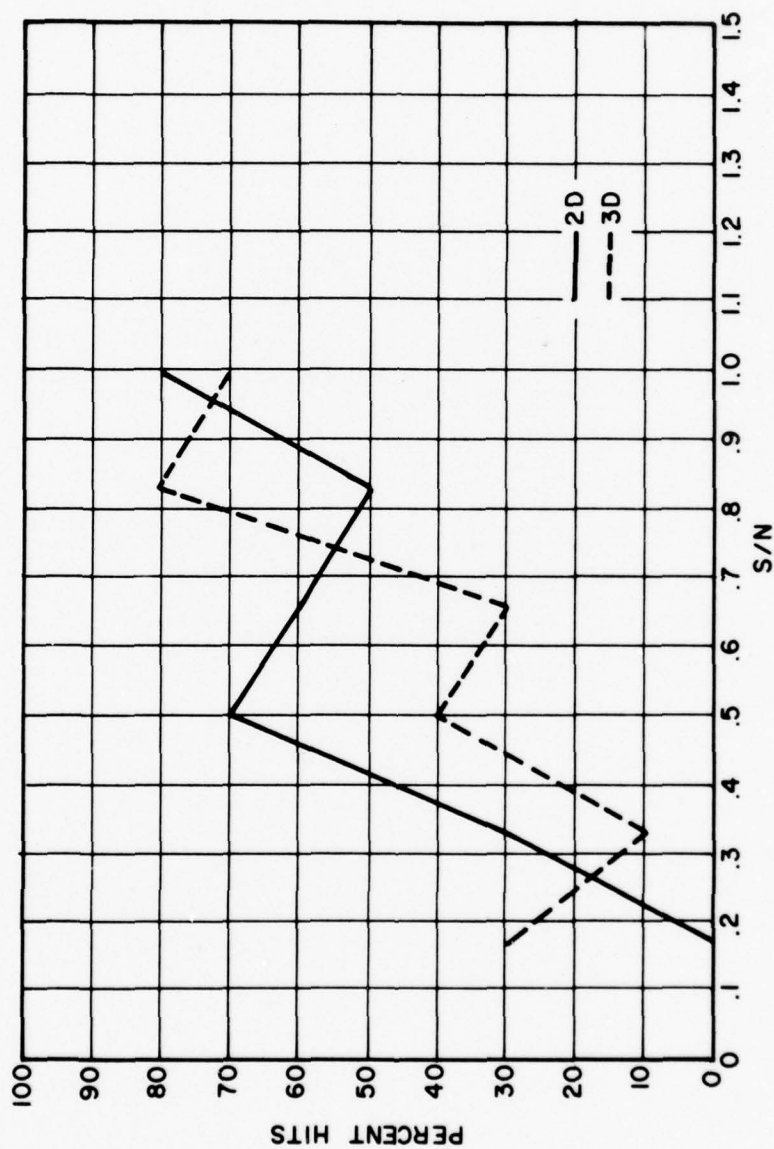


Fig. 6 - COMPARISON OF PERCENT CORRECT RESPONSES AS A FUNCTION OF S/N FOR FLAT AND STEREO DISPLAYS. (NOISE DENSITY=0.30)

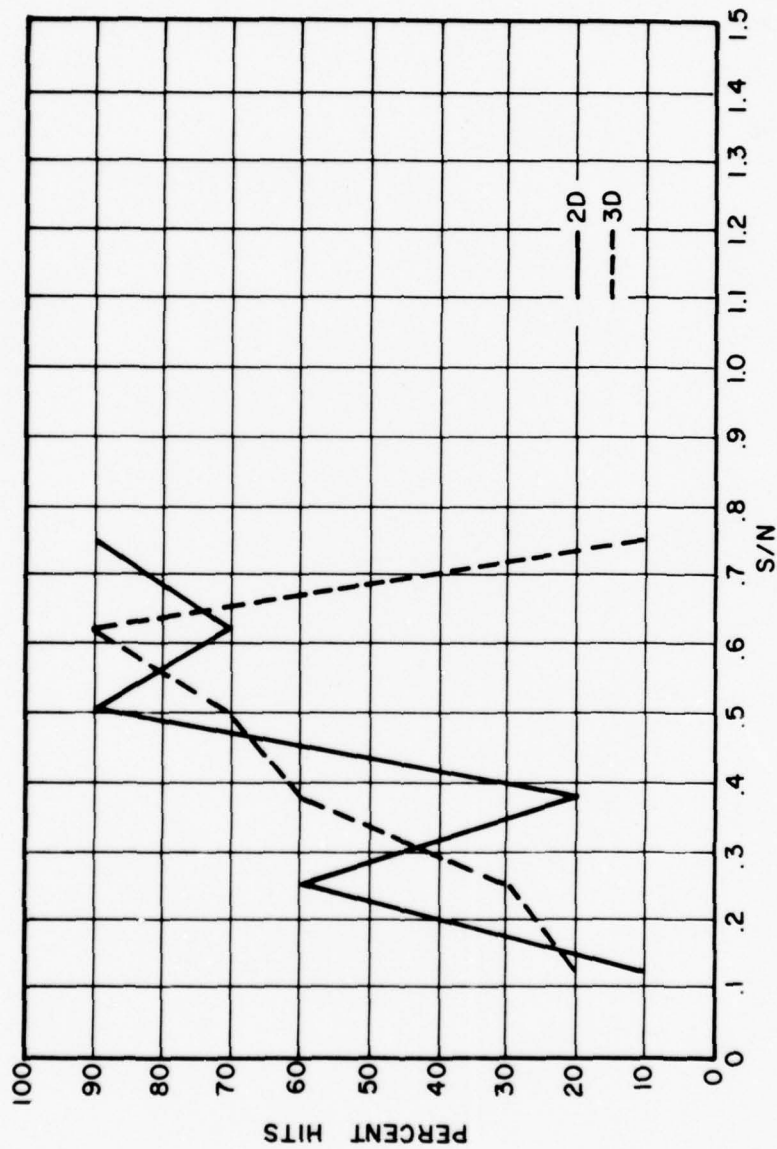


Fig. 7 - COMPARISON OF PERCENT CORRECT RESPONSES AS A FUNCTION OF S/N FOR FLAT AND STEREO DISPLAYS. (NOISE DENSITY = 0.40)

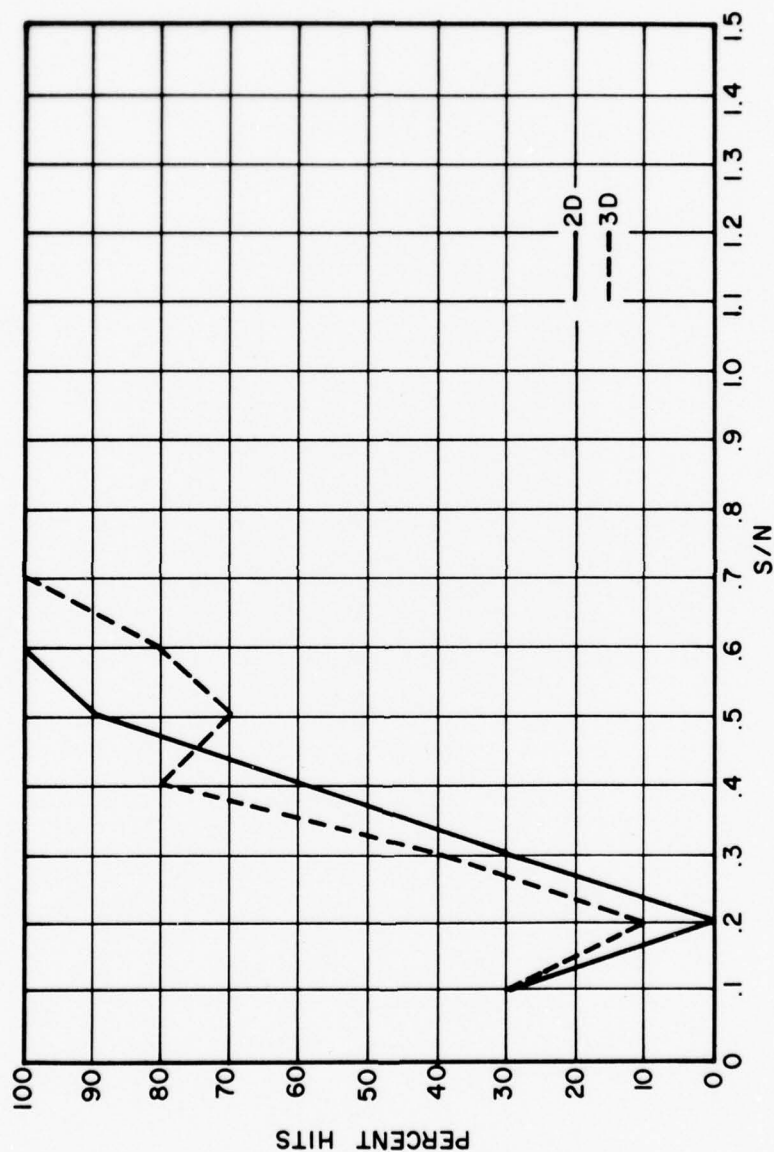


Fig. 8 - COMPARISON OF PERCENT CORRECT RESPONSES AS A FUNCTION OF S/N FOR FLAT AND STEREO DISPLAYS. (NOISE DENSITY=0.50)

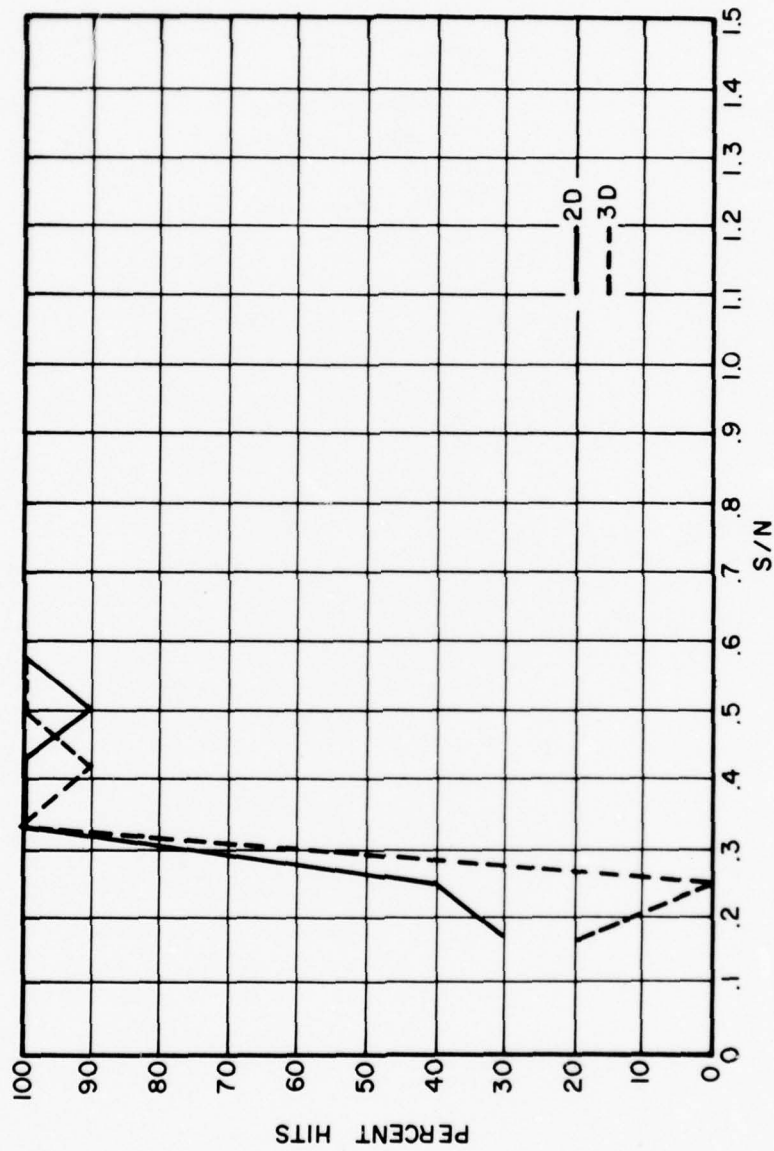


Fig. 9 - COMPARISON OF PERCENT CORRECT RESPONSES AS A FUNCTION OF S/N FOR FLAT AND STEREO DISPLAYS. (NOISE DENSITY = 0.60)

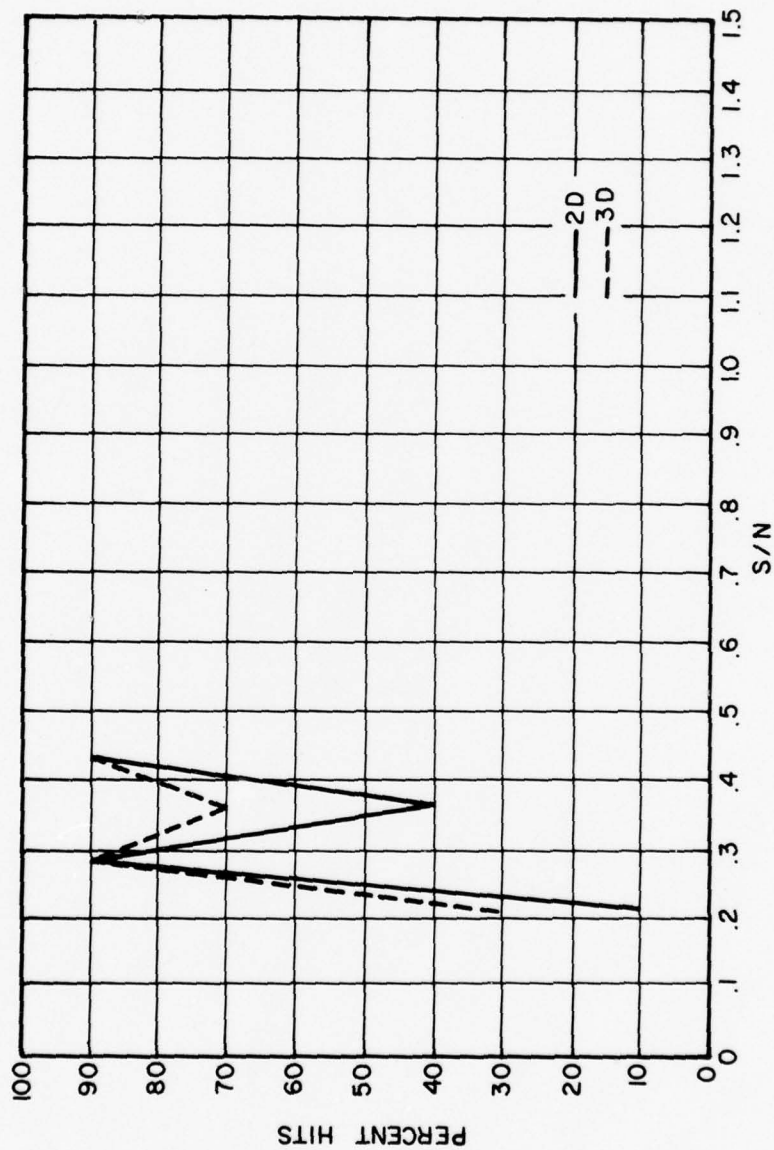


Fig.10 - COMPARISON OF PERCENT CORRECT RESPONSES AS A FUNCTION OF S/N FOR FLAT AND STEREO DISPLAYS. (NOISE DENSITY=0.70)

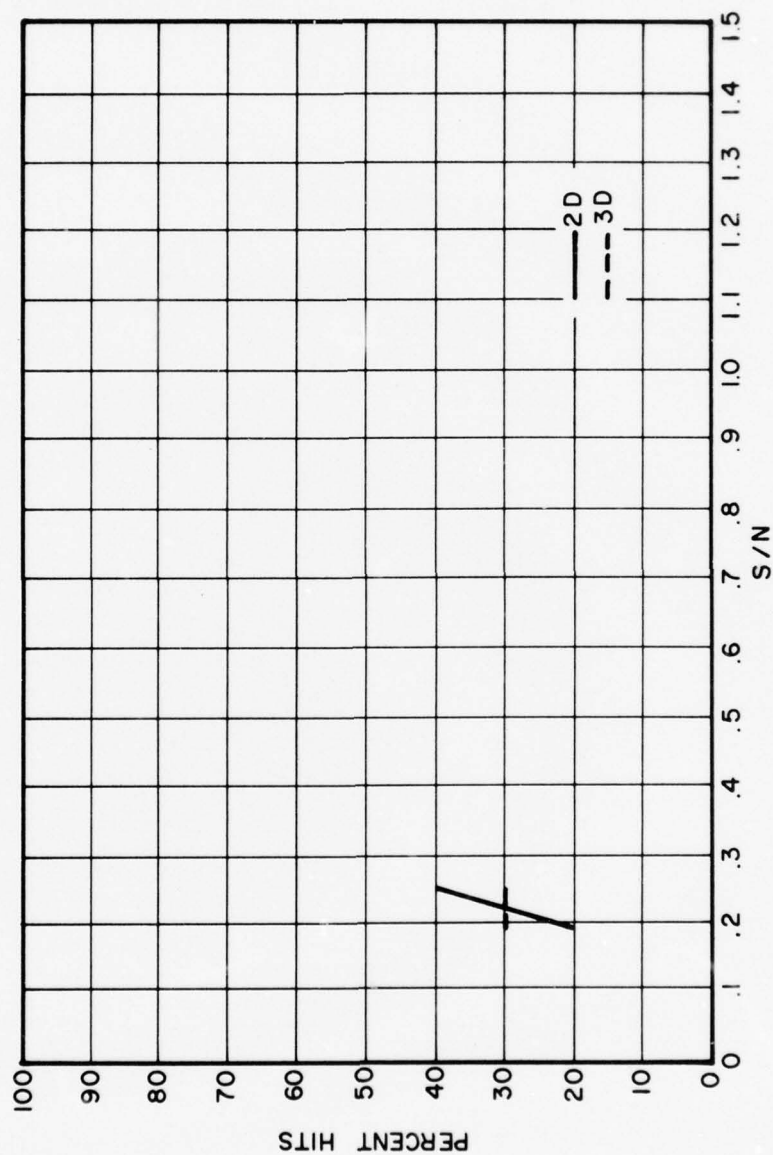


Fig.11 - COMPARISON OF PERCENT CORRECT RESPONSES AS A FUNCTION OF S/N FOR FLAT AND STEREO DISPLAYS. (NOISE DENSITY=0.80)

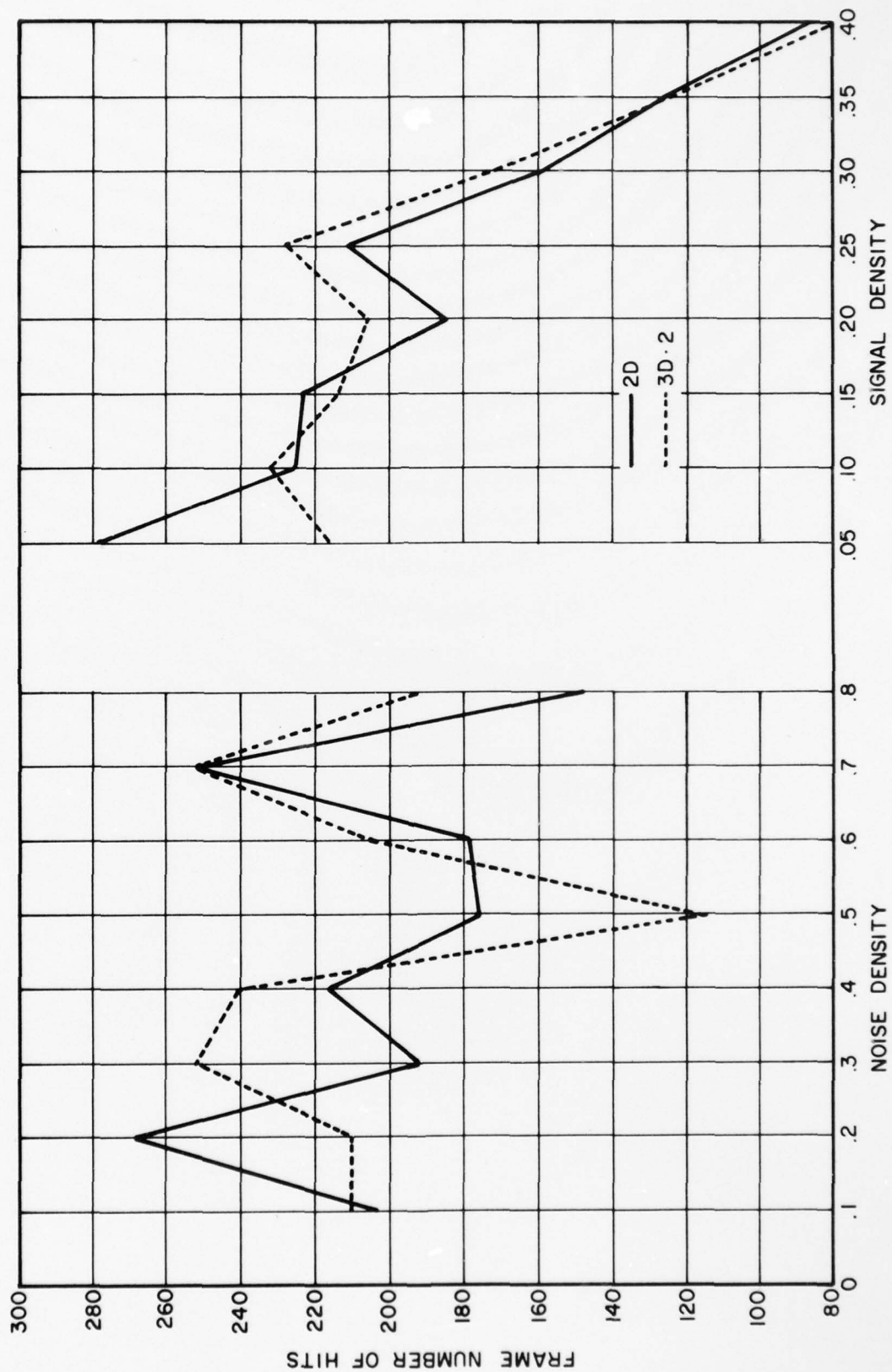


Fig. 12 - COMPARISON OF FRAME NUMBER OF HITS FOR FLAT AND STEREO DISPLAYS

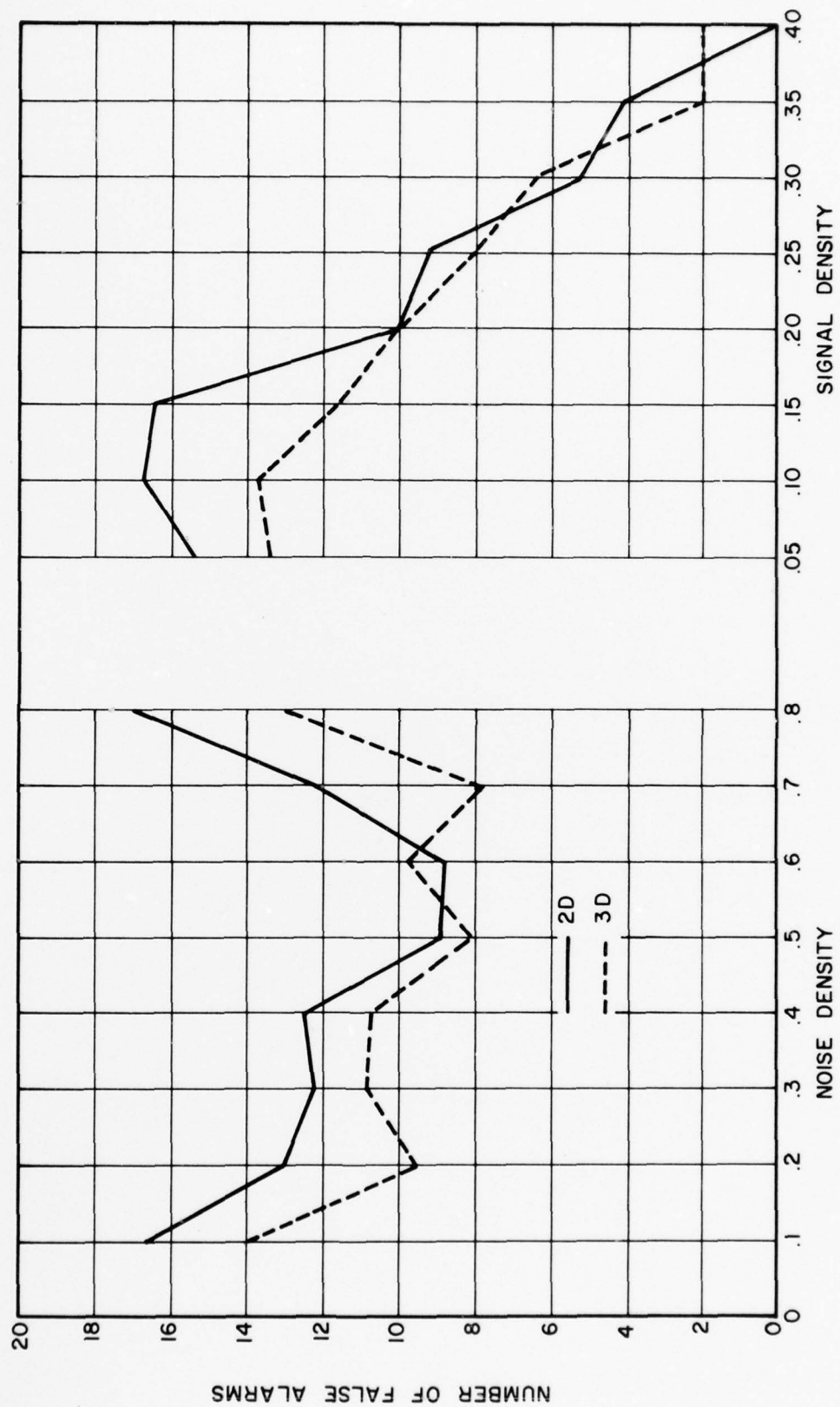


Fig. 13 - TOTAL NUMBER OF FALSE ALARMS FOR FLAT AND STEREO DISPLAYS

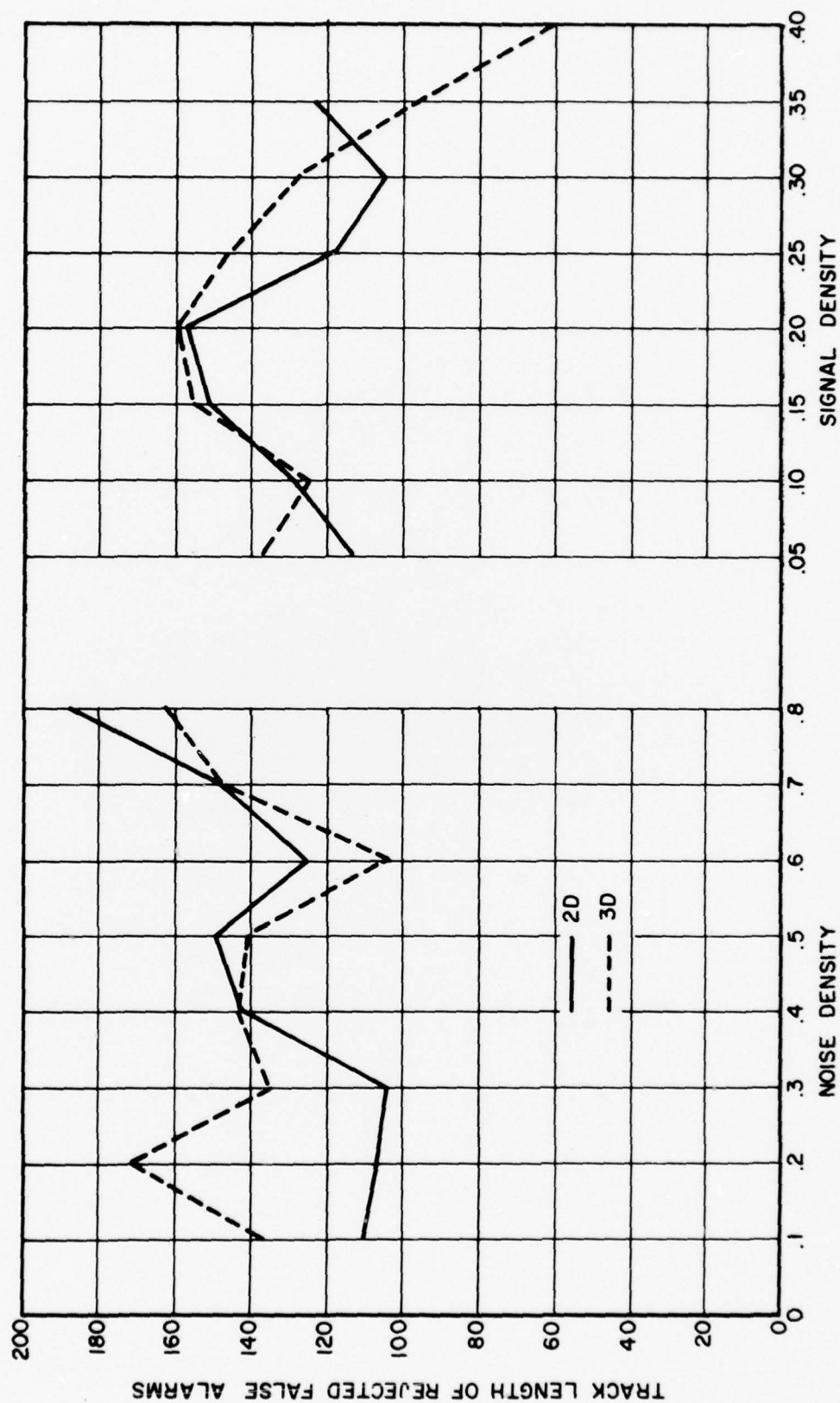


Fig. 14 - COMPARISON OF FALSE ALARM TRACK LENGTHS FOR FLAT AND STEREO DISPLAYS

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APPENDIX
to
SUMMARY REPORT
Problems 8 and 13
NObsr 85185

In our original proposal to assess the usefulness of SCENICS, a stereoscopic (3-D) display, we showed some tentative calculations of an advantage we might expect to get from the use of the technique, as opposed to the conventional flat (2-D) display. They are repeated here unaltered except for the form of the notation.

Let p = probability of marking by noise

and $p' = p + \Delta p$, where Δp is the probability of marking by signal.

In general

$$\frac{(p + \Delta p)^2}{p^2} > \frac{p + \Delta p}{p}$$

$$[\text{Stereo}] > [\text{Flat}]$$

An estimate of the advantage expected, A.E., of the stereo display vs. the flat display may be had by subtracting the value of

$$\frac{p + \Delta p}{p} \text{ from } \frac{(p + \Delta p)^2}{p^2}, \text{ thus, A. E.} = \frac{(p + \Delta p)^2}{p^2} - \frac{p + \Delta p}{p} =$$

$$\frac{\Delta p^2 + p \cdot \Delta p}{p^2}.$$

Figure 3 of this appendix shows a family of curves obtained by substituting the indicated p and Δp values into the equation.

What may we conclude from this figure? For one thing, as $p \rightarrow 0$, the expected advantage, A. E., goes to infinity, and at small values of p and large values of Δp , A. E. is large. But the result is trivial: even without an advantageous display large, strong signals are easy to detect. Our real interest is in the situation in which we can increase the gain, let in more noise, and thus, weak signals.

Dr. A. F. Wittenborn has examined this problem and he has written a TRACOR internal memorandum concerned with it. He has analyzed the problem exactly as the experiment was performed, which, as was pointed out earlier, suffered from "throwing away" half the potential pairs of marks. The memorandum follows. Note that Figure 1 has values different from those mentioned in the memorandum. Table IV has been added to show the values plotted in Figure 1.

August 19, 1963

MEMORANDUM

To: B. H. Deatherage
From: A. F. Wittenborn
Subject: SCENICS--A Brief Analysis Thereof

This memorandum describes an analysis of the SCENICS technique for displaying bearing-time or other data in stereo form and compares the results which one expects from this method of display to the results which one expects from a conventional flat display. The analysis shows why the SCENICS display and the flat display produced essentially identical results in the psychophysical experiment you carried out.

Consider the manner in which the flat display was created. A particular bearing-time location contains either a mark or it is not marked. The probability of marking due to noise alone is p . Let the total number of trials for marking or not marking be m . Along any possible path, the expected number of marks, E_n , due to noise alone is, since the marking process is a game following the Bernoulli distribution

$$E_n = mp$$

and the standard deviation σ_n about this value is

$$\sigma_n = \sqrt{mp(1 - p)}$$

When a signal is present, the marking probability is p' , where $p' \geq p$, (the equality is evidently a situation of academic interest only, in order to determine limit behavior). The expected number of marks for signal and noise is then

$$E_{n+s} = mp'$$

Following the usual procedure used for calculating output signal-to-noise ratio, we define

$$\left(\frac{S}{N}\right)_f = \frac{E_{n+s} - E_n}{\sigma_n},$$

i.e. the output signal-to-noise ratio for the flat display is defined to be the ratio of the increase in mean number of marks due to signal and the standard deviation about the mean number of marks due to noise alone. Thus

$$\left(\frac{S}{N}\right)_f = \frac{\sqrt{m}(p' - p)}{\sqrt{p(1 - p)}}$$

The SCENICS display is equivalent to considering only those marks along a possible target track in the flat display which occur in succession. More specifically, due to the manner in which the stereo display must be constructed, only every other possible pair can appear in the reference or signal plane of the display. The output signal-to-noise ratio for the SCENICS display, defined as for the flat display, is therefore

$$\left(\frac{S}{N}\right)_s = \sqrt{\frac{m}{2}} \frac{(p'^2 - p^2)}{\sqrt{p^2(1 - p^2)}}$$

Define a ratio R as

$$R = \left(\frac{S}{N}\right)_s / \left(\frac{S}{N}\right)_f$$

$$= \frac{1}{\sqrt{2}} \frac{(p'^2 - p^2)}{\sqrt{p^2(1 - p^2)}} \frac{p(1 - p)}{(p' - p)}$$

$$= \frac{(p' + p)}{\sqrt{2p(1 + p)}}$$

R is a measure for comparing a SCENICS display to its flat counterpart.

Tables I, II, and III give the values of $(S/N)_f \frac{1}{\sqrt{m}}$ and $\frac{1}{\sqrt{m}} \left(\frac{S}{N}\right)_s$ and R as a function of p and p', with $p \leq p' \leq 1$. The data of Table III for $R(p, p')$ are shown graphically in Figure 1. Note that R can be greater or less than unity, depending on the values of p and p'. R is plotted, by the solid lines, as a function of p for constant (p' + p). The dotted lines give R as a function of p for constant p'. The dashed lines show R as a function of p for constant $\left(\frac{p' - p}{p}\right)$, which is a measure of the input signal-to-noise ratio.

Note that the analysis, to this point, suggests that the SCENICS technique will produce poorer output signal-to-noise ratios than the flat display for a number of physically very interesting situations. Consider why this is so. In much of your previous writing concerning SCENICS, you have mentioned signal enhancement. For example, if $p = .1$ and $p' = .2$, then

$$\frac{p' - p}{p} = 0.2 - 0.1$$

$$= 1.0$$

For the corresponding situation in the SCENICS display

$$\frac{p'^2 - p^2}{p} = 0.04 - 0.01$$
$$= 3.0$$

This suggested enhancement of the relative marking density due to signal is, in fact, real. The difficulty, however, arises when "the story" is completed. In considering signal enhancement only, one has not taken into account the standard deviation in the noise marking density. Note that as E_n decreases, the relative value of σ_n compared to E_n increases. Thus, when the total expected number of marks is decreased, which it is in the SCENICS technique, the signal enhancement suggested above is offset by an increase in the variability of the noise background.

It is of interest to consider the actual experimental data you obtained in your experiment and see how it fits with the treatment given above. First of all, one might say that on the basis of Figure 1 the SCENICS results should have perhaps been inferior to those obtained with the flat display. This is not true, for the following reason: It is often stated that a signal can be detected in a noise background 50% of the time if the signal-to-noise ratio is some 9 db or a factor 3.

A. F. Wittenborn

lr

An objection to A. F. Wittenborn's statement on page 2 of the memorandum, "manner in which the stereo display must be constructed," must be entered. The display need not be constructed as it was, but rather in the new way, described in the body of the report, to take advantage of all possible pairs of marks. The new method retains independence between left and right eye views, similarly to the original method. It does introduce some dependence in that lines presented to a single eye are each repeated once. The psychological import of this dependence, it must be said, is unknown.

What is important is that the function R is changed to

$$R = \frac{p' + p}{\sqrt{p(1 + p)}}$$

which shifts the plot upward by a factor of $\sqrt{2}$. Figure 1 is actually plotted in accordance with the latter values of R. The liberty of changing A. F. Wittenborn's Fig. 1 was taken because it will be used in this revised form.

For Fig. 2, R is set equal to 1, thus

$$\frac{p' + p}{\sqrt{p(1 + p)}} = 1$$

and solved for p' and thus Δp can be ascertained for those stereo cases which are always better than flat. There are some other implications which we shall now examine.

The accepted method for computing signal-to-noise ratios when the quantities are amplitudes or voltages is to use $20 \log (\text{Ratio})$, and when the quantities are power or energy use $10 \log (R)$. In this discussion of the relation between stereo and flat displays the quantities are probabilities of marking, which is related to

power, and so we might expect to calculate dB advantage of stereo over flat as $10 \log R =$

$$10 \log \frac{\sqrt{m} \left(\frac{S}{N} \right) S}{\sqrt{m} \left(\frac{S}{N} \right) f} = 10 \log \frac{p' + p}{\sqrt{p(1+p)}}$$

where p = noise marking density or probability of marking from noise alone,

and $p' = p + \Delta p$ = noise-plus-signal marking density or probability of marking from noise plus signal.

We do not know if the "eye" works that way. Does the human observer respond to average densities or powers or by comparing a peak to nearby peaks; that is, amplitudes or voltages. If he, in fact, does the latter we probably ought to compute signal-to-noise ratios as $20 \log R$.

Since the answer is not known we should calculate S/N both ways; then ascertain which most nearly fits the empirical data.

Let us make a few illustrative calculations for the stereo case in which each odd-numbered time-sample is visually matched with both the preceding and succeeding even-numbered time-samples.

Consider 10 lines of bearing-time data, i.e., $m = 10$ and $\sqrt{m-1} = 3$.

NOTE: $m-1$ is preferred when m is small.

Likewise, consider $m = 50$ and $\sqrt{m-1} = 7$.

We may set the gain of our devices to yield $p = 0.7$, i.e., the probability of marking by noise alone equals 0.7.

Now let $\Delta p = .134$, $.2$ and $.3$, therefore $p' = .834$, $.9$ and 1.0 .

(.134 is chosen because for that value of Δp the stereo display is never worse than the flat). Substituting these values appropriately in

$$\left(\frac{S}{N} \right)_s = \sqrt{m-1} \frac{p'^2 - p^2}{\sqrt{p^2(1-p^2)}}, \quad p = .7, m = 9$$

For $p' = .834$, $(\frac{S}{N})_s = 3 \times .400 = 1.20$

and $10 \log (\frac{S}{N})_s = 0.79 \text{ dB}$, or $20 \log (\frac{S}{N})_s = 1.58 \text{ dB}$

For $p' = .9$, $(\frac{S}{N})_s = 3 \times .640 = 1.92$

and $10 \log (\frac{S}{N})_s = 2.83 \text{ dB}$, or $20 \log (\frac{S}{N})_s = 5.66 \text{ dB}$

For $p' = 1.0$, $(\frac{S}{N})_s = 3 \times 1.02 = 3.06$

and $10 \log (\frac{S}{N})_s = 4.86 \text{ dB}$, or $20 \log (\frac{S}{N})_s = 9.71 \text{ dB}$

These are output signal-to-noise ratios in decibels.

Similarly,

$p' = .834$, $(\frac{S}{N})_s = 7 \times .400 = 2.80$

$10 \log 2.80 = 4.5 \text{ dB}$, $20 \log 2.80 = 8.9 \text{ dB}$

$p' = .9$, $(\frac{S}{N})_s = 7 \times .640 = 4.48$

$10 \log 4.48 = 6.5 \text{ dB}$, $20 \log 4.48 = 13.0 \text{ dB}$

$p' = 1.0$, $(\frac{S}{N})_s = 7 \times 1.02 = 7.14$

$10 \log 7.14 = 8.5 \text{ dB}$, $20 \log 7.14 = 17.1 \text{ dB}$

The values for $R = \frac{(\frac{S}{N})_s}{(\frac{S}{N})_f}$ at $p' = .834$, $.9$ and 1.0 ,

$p = .7$ (Note that the factor \sqrt{m} cancels out) are

1.40, 1.47 and 1.56.

$10 \log 1.40 = 1.5 \text{ dB}$, $20 \log 1.40 = 2.9 \text{ dB}$

$10 \log 1.47 = 1.7 \text{ dB}$, $20 \log 1.47 = 3.3 \text{ dB}$

$10 \log 1.56 = 1.9 \text{ dB}$, $20 \log 1.56 = 3.9 \text{ dB}$

The values of R when $(\frac{S}{N})_s = .5$ (which determines p' when values of p are assumed) are all near the center line where $R = 1.414$ in the region $.5 \leq p \leq .7$, the region of best detections. Since p' is determined we can solve for Δp .

$\Delta p = .21, .18$ and $.16$ when $p = .5, .6$ and $.7$ respectively. We should expect, therefore, to be able to detect lower $\frac{\Delta p}{p}$ ratios at the higher value of p

$$\begin{aligned} 10 \log R &= 10 \log 1.414 = 10 \times 0.15 = 1.5 \text{ dB} \\ \text{and } 20 \log R &= 20 \log 1.414 = 20 \times 0.15 = 3.0 \text{ dB} \end{aligned}$$

We might therefore be justified also in stating for noise-alone masking densities between $.5$ and $.7$, that when the output signal-to-noise ratio in the stereo case is a half, the stereo display should be about 3 dB better than the flat display. As a specific instance let $\frac{\Delta p}{p} = \frac{.18}{.60}$, an input signal-to-noise ratio equal to 0.30 . The output signal-to-noise ratio $= 0.50$, and the stereo display is 3 dB better than the flat. The general conclusion is that the stereo case, for those values of p (0.5 to 0.7) showing the best detection rates, is enough better than the flat, even for weak signals and thus low signal-to-noise ratios, that its advantage ought to be capable of exploitation.

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$\frac{p}{p'}$.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0	
.1	0	.302	.804	1.508	2.412	3.518	4.824	6.332	8.040	9.950	
.2		0	.255	.612	1.071	1.633	2.296	3.061	3.929	4.898	
.3			0	.245	.559	.944	1.399	1.923	2.517	3.182	
.4				0	.245	.546	.900	1.309	1.773	2.291	
.5					0	.254	.554	.901	1.293	1.732	
.6						0	.271	.583	.9375	1.333	
.7							0	.300	.640	1.020	.400
.8								0	.312	.750	
.9									0	.484	
1.0										0	

$$\sqrt{\frac{2}{m}} \left(\frac{s}{N} \right)_s = \frac{p'^2 - p^2}{\sqrt{p^2(1-p^2)}}$$

FOR A STEREO DISPLAY

TABLE II - OUTPUT SIGNAL-TO NOISE RATIOS, CALCULATED AS $\sqrt{\frac{2}{N}} \left(\frac{S}{N} \right)_s = \frac{p^2 - p^2}{\sqrt{p^2(1-p^2)}}$, FOR A STEREO DISK

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$\begin{matrix} p' \\ p \end{matrix}$.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0
.1	.428	.637	.848	1.06	1.27	1.49	1.70	1.91	2.04	2.39
.2		.579	.721	.865	1.01	1.15	1.30	1.44	1.59	1.73
.3			.680	.795	.908	1.02	1.13	1.24	1.36	1.47
.4				.758	.824	.915	1.01	1.13	1.23	1.33
.5					.817	.898	.975	1.06	1.15	1.22
.6						.867	.908	.980	1.05	1.15
.7							.896	.975	1.04	1.10
.8								.930	1.00	1.06
.9									.974	1.03
1.0										1.00

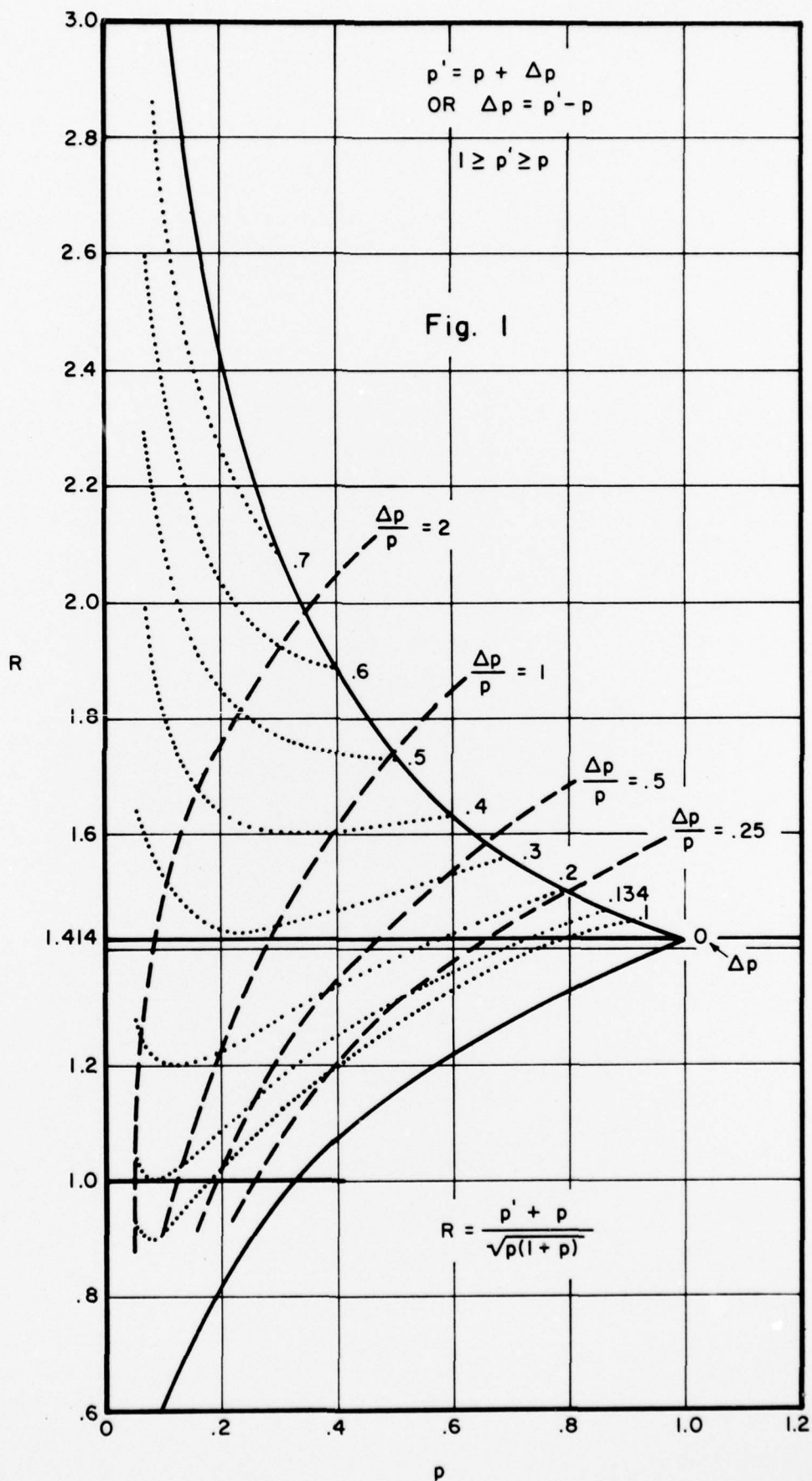
When $p = 1.0$ and $\Delta p = 0$, $R = 1.00$

TABLE III - CELL ENTRIES ARE THE RATIO $R = \frac{(\frac{S}{N})_s}{(\frac{S}{N})_f}$ FOR PAIRED VALUES OF p AND p' ,

WHERE $R = \frac{p' + p}{\sqrt{2p(1 + p)}}$

Restriction: $1 \geq p' \geq p$

[illegible]



$1 \geq p' \geq p$ IS A RESTRICTION

p = NOISE MARKING DENSITY

p' = NOISE - PLUS - SIGNAL MARKING DENSITY

$p' = p + C$ AND FOR $C = 0.134$ STEREO IS ALWAYS BETTER THAN FLAT.

FOR $p \geq 0.333$ STEREO IS ALWAYS BETTER THAN FLAT.

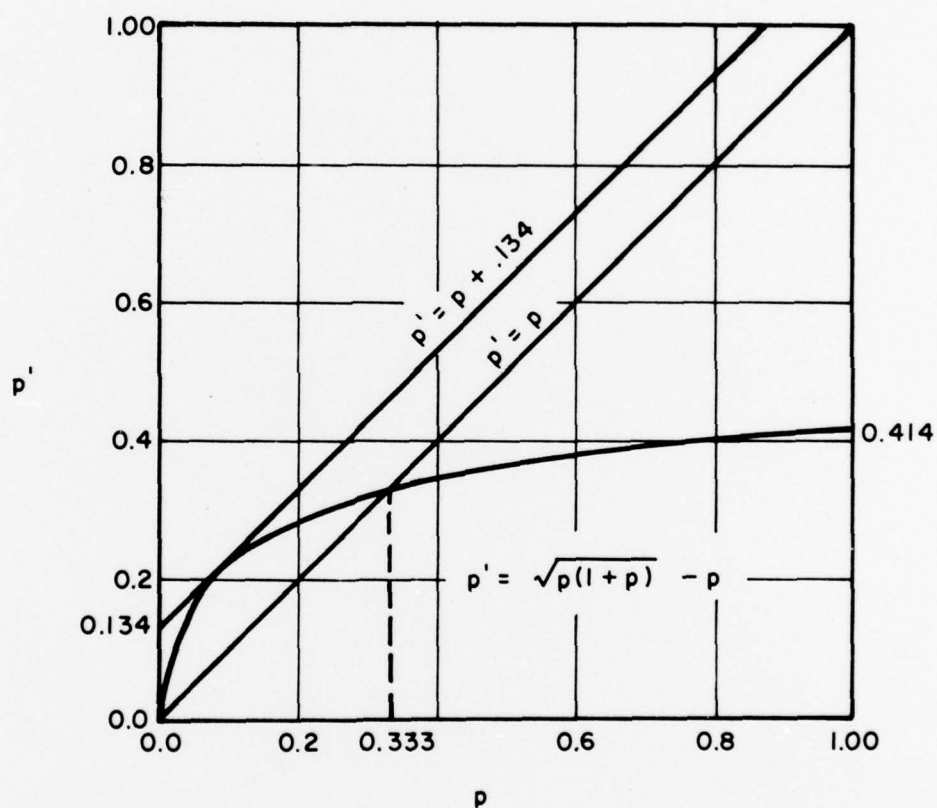


Fig. 2

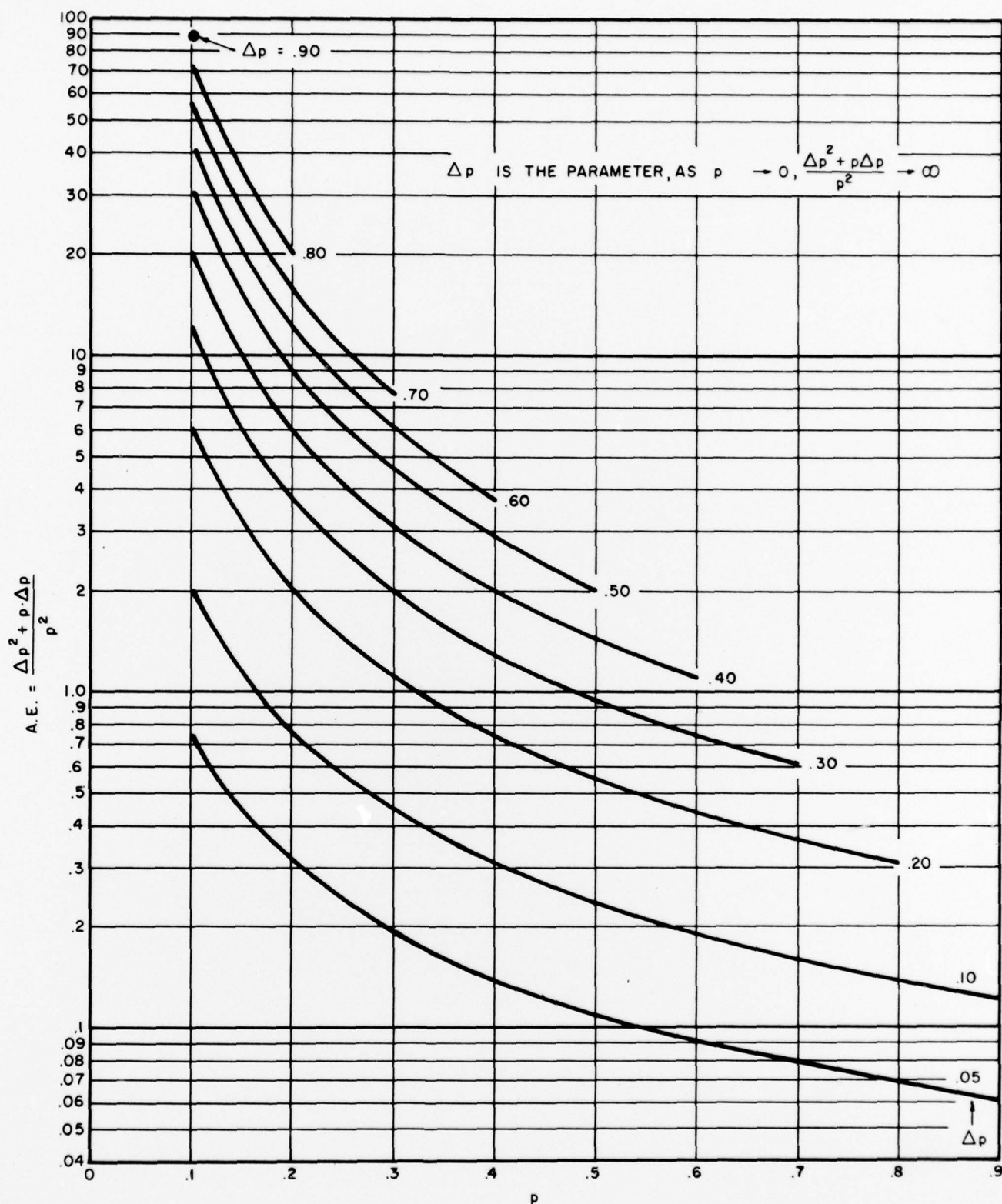


Fig. 3

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